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STABILITY OF CYLINDRICAL SHELLS OF OVAL CROSS SECTION IN THE BENDING STATE OF STRESS

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The stability of cylindrical shells of oval cross section under the combined effect of axial compression and axisymmetric loads is investigated taking account of the bending in the subcritical state. The investigation is based on an analysis of the solution of the equilibrium equations of shells of oval section obtained by using finite differences. The magnitudes of the critical loads are determined by numerical analysis on an electronic digital computer.

1. Let us use the equilibrium and strain compatibility equations for the stress function φ and the radial displacement function ω for a cylindrical shell [1] in the dimensionless form

$$k_0^2\nabla^4\varphi + k(\theta)\frac{\partial^2\omega}{\partial\xi^2} - \left(\frac{\partial^2\omega}{\partial\xi\,\partial\theta}\right)^2 + \frac{\partial^2\omega}{\partial\xi^2}\frac{\partial^2\omega}{\partial\theta^2} = 0 \tag{1.1}$$

$$\frac{1}{12(1-\sigma^2)k_0^2}\nabla^4\omega - k(\theta)\frac{\partial^2\varphi}{\partial\xi^2} - \frac{\partial^2\varphi}{\partial\xi^2}\frac{\partial^2\omega}{\partial\theta^2} + 2\frac{\partial^2\omega}{\partial\xi\,\partial\theta}\frac{\partial^2\varphi}{\partial\xi\,\partial\theta} - \frac{\partial^2\omega}{\partial\xi^2}\frac{\partial^2\varphi}{\partial\theta^2} = 0$$

$$\nabla^4 = \frac{\partial^4}{\partial\xi^4} + 2\frac{\partial^4}{\partial\xi^2\,\partial\theta^2} + \frac{\partial^4}{\partial\theta^4}$$

$$\varphi = \frac{\Phi}{EhR_0^2}, \quad k(\theta) = \frac{R(\theta)}{h}, \quad \xi = \frac{x}{R_0}, \quad \theta = \frac{y}{R_0}$$

to solve the problem. Here ω is the deflection, positive within the shell, and referred to the shell thickness h, Φ is a stress function in the middle surface, and $R(\theta)$ is the shell radius.

Let us investigate cylindrical shells with cross section radius of curvature

$$R(\theta) = \frac{R_0}{1+\mu\cos 2\theta} \qquad (0 \leqslant \mu \leqslant 1)$$

We take the buckling boundary conditions in the form

$$\omega = \frac{\partial\omega}{\partial\xi} = \varphi = \frac{\partial\varphi}{\partial\xi} = 0 \quad \text{(rigid support)}$$

$$\omega = M = \varphi = \frac{\partial\varphi}{\partial\xi} = 0 \quad \text{(hinge support)}$$

2. We represent the functions φ and ω as the sum of two components

$$\varphi = \varphi_0(\xi,\theta) + \varphi(\xi,\theta), \quad \omega = \omega_0(\xi,\theta) + \omega(\xi,\theta) \tag{2.1}$$

Here $\varphi_0(\xi,\theta)$, $\omega_0(\xi,\theta)$ characterize the subcritical state of the shell, and $\varphi(\xi,\theta)$, $\omega(\xi,\theta)$ the increments in these quantities due to buckling.

Substituting (2.1) into the system (1.1) and neglecting second order quantities, we linearize this system to

$$k_0^2\nabla^4\varphi + k_0(1+\mu\cos 2\theta)\frac{\partial^2\omega}{\partial\xi^2} - 2\frac{\partial^2\omega_0}{\partial\xi\,\partial\theta}\frac{\partial^2\omega}{\partial\xi\,\partial\theta} + \frac{\partial^2\omega_0}{\partial\xi^2}\frac{\partial^2\omega}{\partial\theta^2} + \frac{\partial^2\omega_0}{\partial\theta^2}\frac{\partial^2\omega}{\partial\xi^2} = \left(\frac{\partial^2\omega_0}{\partial\xi\,\partial\theta}\right)^2 - \frac{\partial^2\omega_0}{\partial\xi^2}\frac{\partial^2\omega_0}{\partial\theta^2} - k_0(1+\mu\cos 2\theta)\frac{\partial^2\omega_0}{\partial\xi^2} - k_0^2\nabla^4\varphi_0$$

$$\frac{1}{12(1-\sigma^2)k_0^2}\nabla^4\omega - k_0(1+\mu\cos 2\theta)\frac{\partial^2\varphi}{\partial\xi^2} - \frac{\partial^2\varphi_0}{\partial\xi^2}\frac{\partial^2\omega}{\partial\theta^2} - \frac{\partial^2\omega_0}{\partial\theta^2}\frac{\partial^2\varphi}{\partial\xi^2} + 2\frac{\partial^2\omega_0}{\partial\xi\,\partial\theta}\frac{\partial^2\varphi}{\partial\xi\,\partial\theta} + 2\frac{\partial^2\varphi_0}{\partial\xi\,\partial\theta}\frac{\partial^2\omega}{\partial\xi\,\partial\theta} - \frac{\partial^2\omega_0}{\partial\xi^2}\frac{\partial^2\varphi}{\partial\theta^2} - \frac{\partial^2\varphi_0}{\partial\theta^2}\frac{\partial^2\omega}{\partial\xi^2} = \frac{\partial^2\omega_0}{\partial\xi^2}\frac{\partial^2\varphi_0}{\partial\theta^2} - 2\frac{\partial^2\omega_0}{\partial\xi\,\partial\theta}\frac{\partial^2\varphi_0}{\partial\xi\,\partial\theta} + \frac{\partial^2\varphi_0}{\partial\xi^2}\frac{\partial^2\omega_0}{\partial\theta^2} + k_0(1+\mu\cos 2\theta)\frac{\partial^2\varphi_0}{\partial\xi^2} - \frac{1}{12(1-\sigma^2)k_0^2}\nabla^4\omega_0$$

The system (2.2) affords a possibility of finding the solution of the nonlinear system (1.1) for a fixed value of the load parameter, as well as of resolving the question about the stability of the state under consideration. Externally, the right sides of the system (2.2) agree exactly with the left sides of the system (1.1). Hence, when the exact values of the functions φ_0, ω_0 have been found (by any method), the system (2.2) becomes homogeneous. For some value of the load parameter it can have a nontrivial solution.

According to [2], let us take some functions φ_0°, ω_0° as the zero approximation for the numerical solution of (1.1), and let us find the solution with previously assigned accuracy in their neighborhood. The next step is to find the increments φ', ω' which are assumed small

$$\varphi_0 = \varphi_0^\circ + \varphi', \quad \omega_0 = \omega_0^\circ + \omega' \qquad (2.3)$$

Substituting (2.3) into (1.1) and discarding second order quantities in φ' and ω', we obtain a system of linear differential equations (2.2) for φ' and ω'. We append the increments found for the first approximation to the zero approximation solutions and we obtain a new approximation, etc. Let us use finite differences to find the solution of the system (2.2). The successive approximation process can be continued until such solutions are obtained for which the right sides in the system (2.2), replaced by finite-difference equations, would be arbitrarily close to zero.

The differential equations (2.2) are satisfied at the ith node by using difference approximations of the form

$$\frac{1}{2\Delta a}(b_{i+1} - b_{i-1})$$

Here $b = \{\varphi, \omega\}$, Δa is the spacing along the generator or the arc. Then, Eqs. (2.2) become the following finite-difference expression (in vector form):

$$(2.4)$$

$$E_1\varphi_{m-2} + A\varphi_{m-1} + B\varphi_m + A\varphi_{m+1} + E_1\varphi_{m+2} + D_1\omega_{m-1} + Z\omega_m + D_2\omega_{m+1} = F_{0m}$$
$$E_2\omega_{m-2} + X_1\omega_{m-1} + Y\omega_m + X_2\omega_{m+1} + E_2\omega_{m+2} + D_1\varphi_{m-1} + Z\varphi_m + D_2\varphi_{m+1} = H_{0m}$$
$$\varphi_m = \{\varphi_{m0}, \varphi_{m1}, \varphi_{m2}, \ldots, \varphi_{mn}\}$$
$$\omega_m = \{\omega_{m0}, \omega_{m1}, \omega_{m2}, \ldots, \omega_{mn}\}$$

The form of the matrices $E_i, A, B, D_i, Z, X_i, Y, F_{0m}, H_{0m}$ $(i = 1, 2)$ is not presented because of their unwieldiness.

The eight boundary conditions presented above are also written in finite differences at the boundary nodes. The equations at the nodes preceding the boundary are written taking account of the boundary conditions used, and symmetry is taken into account in the equations on the axes of symmetry. The solution of linear equations under the accepted boundary conditions of the subcritical state is selected as the zero approximation. A program is compiled on the BESM-4 computer and it permits determination of the critical loads. Questions of convergence are investigated.

3. Let us examine the problem of stability of a finite cylindrical shell of oval section, loaded by:

1) an annular concentrated force Q applied along the middle of the transverse section,

2) an axial compressive force T,

3) the combined effect of axial compression T and the concentrated stress resultant Q,

4) the combined effect of axial compression T and external pressure P.

For $\sigma = 0.3$ it has been ascertained as a result of computations that the critical value of the load is determined from the expressions

$$Q^* = C_1(\mu)\, Eh\left(\frac{h}{R_0}\right)^{3/2}, \qquad T^* = C_2(\mu)\, Eh\,\frac{h}{R_0}$$

$$P^* = q^* E\left(\frac{h}{R_0}\right)^2, \qquad q^* = C_3(\mu)\,\frac{R_0}{L}\left(\frac{h}{R_0}\right)^{1/2}$$

For a sufficiently long shell in the case of the separate effect of a concentrated force and axial compression, we obtain

$$\begin{array}{lllll} \mu = 0, & 0.01, & 0.1, & 0.15, & 0.2 \\ C_1 = 0.35, & 0.34, & 0.31, & 0.30, & 0.26 \\ C_2 = 0.51, & 0.50. & 0.46, & 0.43, & 0.41 \end{array}$$

Therefore, the critical loads are reduced substantially in comparison with the corresponding load for a circular cylinder [3] as μ increases.

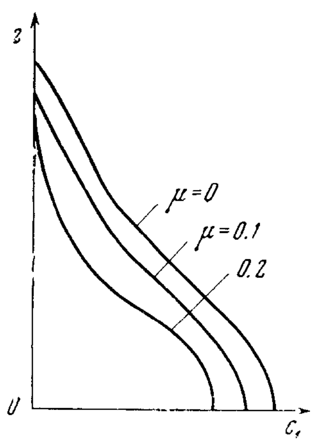

Fig. 1

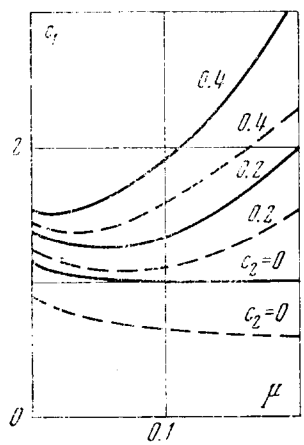

Fig. 2

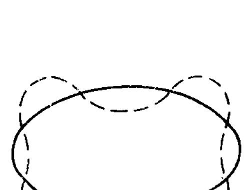

Fig. 3

In the case of the combined effect of a concentrated force and axial compression, the dependence $C_2(C_1)$ is represented in Fig. 1 for fixed μ. The results of investigating the stability of hinge supported (dashes) and clamped (solid lines) shells under the combined effect of axial force T and external pressure P are presented in Fig. 2.

Therefore, an increase in the axial compressive force T contributes to a rise in the critical external pressure P. The shape of the shell waviness under buckling is represented in Fig. 3.

In conclusion, the author thanks I. I. Vorovich for formulating the problem and discussing the results.

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INFLUENCE OF THE SIGN OF SHELL CURVATURE ON THE CHARACTER OF THE STATE OF STRESS

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Problems which turn out to be incorrect in the membrane formulation are investigated. The purpose of this paper is to show that the known anomaly, noted by Vlasov [3], in the behavior of shells of negative curvature and caused by the incorrectness of the formulation of the complete membrane problem for them, is not especially intrinsic property of shells of negative curvature and is observed also in shells of positive curvature, if the complete membrane problem turns out to be incorrect for them. The properties of the stress-strain state are studied as a function of the sign of the middle surface curvature and the manner of edge clamping. The state of stress of the shell is compared with the fundamental state of stress; the edge effect stresses are not taken into account. Two versions of the boundary conditions are considered: one edge of the shell free and the other rigidly clamped (cantilevered shell), and the case when both edges are rigidly clamped.

1. Let us start from the equations and formulas of the bending theory in investigating the state of stress of a thin elastic shell

$$\frac{1}{A}\frac{\partial T_1}{\partial \alpha}+\frac{1}{AB}\frac{\partial B}{\partial \alpha}(T_1-T_2)+\frac{1}{B}\frac{\partial S}{\partial \beta}+\frac{2}{AB}\frac{\partial A}{\partial \beta}S-\frac{N_1}{R_1}+X=0 \quad (\alpha\beta) \qquad (1.1)$$

$$\frac{T_1}{R_1}+\frac{T_2}{R_2}+\frac{1}{AB}\left[\frac{\partial}{\partial \alpha}(BN_1)+\frac{\partial}{\partial \beta}(AN_2)\right]+Z=0$$

$$\frac{1}{A}\frac{\partial G_1}{\partial \alpha}+\frac{1}{AB}\frac{\partial B}{\partial \alpha}(G_1-G_2)-\frac{1}{B}\frac{\partial H}{\partial \beta}-\frac{2}{AB}\frac{\partial A}{\partial \beta}H-N_1=0 \quad (\alpha\beta) \qquad (1.2)$$